Study on Predictive Control for Chiller Plants Based on T-S Fuzzy Model

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SUMMARY
HVAC energy consumption accounts for more than half of the building’s total energy consumption, and the water system energy consumption in the entire HVAC system energy consumption accounted for in the range of 30% to 80%, which has a large energy-saving space. This paper puts forward a chiller plants’ COP prediction model based on the T-S fuzzy model. By using this model, we can easily predict the chiller plants’ COP and keep the COP in a good level by some ahead control strategies. The verification result shows that the model can predict the COP perfectly and can be used in chiller plants’ global control.

INTRODUCTION
The energy consumption of HVAC system is quite heavy, which accounts for more than 50% of building energy consumption. And in the HVAC system, the energy consumption is dominated by HVAC water system, whose energy consumption can account for more than 30%, the worst can up to 80% of HVAC system’s energy consumption. Some studies have done on the HVAC system energy saving, some studies are focused on the water system, the energy consumption can account for more than 50% of building energy consumption, and in the HVAC system, the energy consumption accounts for more than half of the building’s total energy consumption, and the water system energy consumption in the entire HVAC system energy consumption accounted for in the range of 30% to 80%, which has a large energy-saving space. This paper puts forward a chiller plants’ COP prediction model based on the T-S fuzzy model. By using this model, we can easily predict the chiller plants’ COP and keep the COP in a good level by some ahead control strategies. The verification result shows that the model can predict the COP perfectly and can be used in chiller plants’ global control.

MODEL STRUCTURE AND MODELING METHODS
In this paper, we are going to use the T-S fuzzy model to do some parameter analysis to optimize the global control strategy of tower-chiller-pump. The T-S fuzzy model is a kind of fuzzy modeling methods with obvious advantages, which was proposed by Takagi and Sugeno in 1985. It can easily transform a non-linear system into a partial linear system, combined with the PID control or some self-adaptation control it can do the performance evaluation and prediction for the control systems. In the chiller plants, there are many non-linear characteristics due to the system hysteresis and coupling nature. So we can use the T-S model to deal with these non-linear characteristics and simplify the mathematical model. By comparing the prediction results of system performance among the adjacent operating conditions, we can determine the optimal variables’ trends to serve the chiller plants control system in a better way.

Introduction for T-S model
The figure 1 is the neural network structure of T-S fuzzy model, in which \( \mu_j(j = 1, 2, \cdots k; i = 1, 2, \cdots n) \) is the membership degree of the function, \( a_j \) is the adapting degree we obtain after the input of \( x_j \) for each rule, it can be expressed as follows:

\[
a_j = \mu_{A_1}(x_1) \wedge \mu_{A_2}(x_2) \cdots \wedge \mu_{A_k}(x_k)
\]

Or

\[
a_j = \mu_{A_1}(x_1)\mu_{A_2}(x_2) \cdots \mu_{A_k}(x_k)
\]

The output for this fuzzy neural network can be expressed as follows:

\[
y_i = \frac{\sum_{j=1}^{k} a_j y_{ij}}{\sum_{j=1}^{k} a_j} = \sum_{j=1}^{k} \overline{a}_j y_{ij}
\]

In reality, the T-S fuzzy model can indicate the complicated nonlinear system as many linear formulas based on multi-fuzzy rules, specific content as follows:

If we use \( x = [x_1, x_2, \cdots x_n]^T \) to express the antecedent parameters, which consists of multiple variables, and assume \( A_j(j = 1, 2, \cdots k; i = 1, 2, \cdots n) \) is the fuzzy subset of antecedent, then the \( i^{th} \) fuzzy rule \( R_i \) can be expressed as follows:

\[
\text{if } x_k \text{ is } A_{k_1} \text{ and } x_n \text{ is } A_{k_1} \quad \text{then } y^i = p_0 + p_1 x_1 + \cdots + p_n x_n = \sum_{m=0}^{n} p_m x_m
\]

In this expression, the parameters \( p_0 \sim p_n \) are constants of consequent parameters.

![Figure 1. The Neural Network Structure of T-S fuzzy Model](image-url)
The antecedent network consists of the antecedent parameters and variables and the consequent network consists of the consequent parameters and variables are two important component of the T-S fuzzy neural network. The different parts of the antecedent network are respectively corresponding with input component \( x_i \), membership degree \( \mu_i \), adapting degree \( a_i \) and the results of adapting degree uniformization \( \bar{a}_i \) in figure 1; the first layer of the consequent network is corresponding with the input component of antecedent parameters \( x_1, x_2, \ldots, x_n \) and \( x_n = 1 \), the last two layers of the consequent network are respectively corresponding with connection weight \( y_{ij} \) and output of each sub-network \( y_i \).

**Double clustering T-S model**

Cluster analysis, a multivariate statistics method for sample classification, can be classified into two different kinds of clusters named R-Cluster and Q-Cluster. R-Clustering analysis is to classify variables and Q-Clustering analysis is to classify samples. In this paper, in order to make a further analysis and propose a better control strategy for both chiller water side and cooling water side, the Q-Clustering analysis will be used on the temperature difference between the chiller water supply and return, the condensing pressure and the COP of chiller plants.

Here, we are going to introduce the double clustering T-S model for the sample data analyzing. In this identification method, six steps should be followed:

1. Determine the fuzzy rule numbers of the optimal cluster result by trial.
2. Analyze the sample data by fuzzy clustering method, and determine the cluster center and the membership of data.
3. Acquire the consequent parameter space by WRLS (Weighted Recursive Least Squares).
4. Take the fuzzy clustering on the consequent parameter space and get the new consequent parameters.
5. Update the membership of antecedent parameters based on the new consequent parameters, and identify the antecedent parameters. The membership function is expressed by the bell-type function expressed by Gaussian function:

\[
A(c, \delta) = \exp \left\{ -\left( \frac{x-c}{\delta} \right)^2 \right\} \tag{5}
\]

Of which \( c \) is the center point of the bell-type membership function, \( \delta \) is the shape coefficient of the bell-type membership function.

6. Calculate the trimming of antecedent parameters and consequent parameters by gradient descent method and output the modeling result. The expression of consequent is the same as the expression (4).

The steps 2~5 mentioned above are the processes of double clustering algorithm rough adjustment, its specific process can be illustrated by follow pictures:

**Figure 2. The roughing process in double clustering algorithm**

For better evacuation on the model identification precision, the absolute relative errors and the parameter \( \text{PER} \) both can be the evaluation criterion, and the \( \text{PER} \) can be expressed as follows:

\[
\text{PER} = \left[ \frac{1}{m} \sum_{k=1}^{m} (y(k) - \hat{y}(k))^2 \right]^{1/2} \tag{6}
\]

In this expression, \( y(k) \) is the \( k^{th} \) sample data, \( k = 1, 2, \cdots, m; \hat{y}(k) \) is the model predicting values.

Two clustering have done in this identification process. The first clustering is for the data cluster, so we can get the membership of sample data. The second clustering is for the consequent parameter space, so we can simplify the consequent parameters to make the model easy to solve.

**IDENTIFICATION OF THE PREDICTION MODEL ON CHILLER PLANTS**

According to the correlation analysis result of the sample data, we found that the temperature difference of chiller water supply and return \( \Delta \) and the condensing pressure \( P_{\text{con}} \) were obviously related to the system COP. So, here we use \( \Delta \) and \( P_{\text{con}} \) as the antecedent parameters, expressed as \( x = [\Delta P_{\text{con}}]^T \), and system COP as the consequent parameter, assume that \( A_j (j = 1, 2, \cdots; k = 1, 2, \cdots) \) is the antecedent fuzzy subset. The \( i^{th} \) fuzzy rule \( R_i \) of COP prediction model can be expressed as follows:

\[
\text{if } \Delta t = A_{1j}, P_{\text{con}} = A_{2j} \text{ then } C_{OP} = p_0 + p_1 \Delta t + p_2 P_{\text{con}} \tag{7}
\]

Next step, we get the valid sample data size of 1874, in which we select the sample data size of 1200 to identify the model, and group the data into four part by the range of temperature and humidity, as shown in figure 3.
Figure 3. The distribution of identification data’s temperature and humidity

Follow the identification steps of T-S fuzzy model, we can find that the cluster number (rule number) with best identification precision is 6. Figure 4 and figure 5 show the renderings with clusters and cluster center in 2D and 3D after the first fuzzy clustering.

Figure 4. The clustering results of 1# unit local operating parameters

As the figures show above, compare to the system COP and the temperature difference between chiller water supply and return, the condensing pressure has the obvious impact on the clustering hierarchy. That means the change of condensing pressure will have much more impact on the model, so we can use the condensing pressure to trim the model.

The text below will give a specific illustration for the prediction model of chiller plants, meanwhile, we select the test data size of 590 from the rest of sample data to verify model. The clustering data distribution is show in table 2.

We can learn from the steps of double clustering algorithm that the rough adjustment is enough for model identification. And in order to verify the necessity of trim, we have made a comparison on the prediction results between no trim and trim. That is show in figure 6 and figure 7. From the figure we can see that, the average relative error between the prediction results and the sample value is 14.48% (maximum relative error is 34.2%, minimum relative error is 0.02%) before the trim, and the PER is 0.8594. After we do the trim, the average relative error between the model and the sample is 13.68% (maximum relative error is 33.09%, minimum relative error is 0.05%), and the PER is 0.8047.

Table 1. The information of identification data

<table>
<thead>
<tr>
<th>Data Zone</th>
<th>Zone I</th>
<th>Zone II</th>
<th>Zone III</th>
<th>Zone IV</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>26.5℃~31℃</td>
<td>31℃~35.8℃</td>
<td>29℃~34℃</td>
<td>34℃~38.6℃</td>
<td>26.5℃~38.6℃</td>
</tr>
<tr>
<td>RH</td>
<td>65%~93%</td>
<td>65%~93%</td>
<td>40%~65%</td>
<td>40%~65%</td>
<td>40%~93%</td>
</tr>
<tr>
<td>Data Size</td>
<td>324</td>
<td>271</td>
<td>320</td>
<td>285</td>
<td>1200</td>
</tr>
<tr>
<td>Data ratio</td>
<td>27%</td>
<td>22.58%</td>
<td>26.67%</td>
<td>23.75%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 2. The clustering data distribution

<table>
<thead>
<tr>
<th>Clusters</th>
<th>Cluster 1</th>
<th>Cluster 2</th>
<th>Cluster 3</th>
<th>Cluster 4</th>
<th>Cluster 5</th>
<th>Cluster 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identification Data</td>
<td>194</td>
<td>179</td>
<td>217</td>
<td>232</td>
<td>285</td>
<td>93</td>
</tr>
<tr>
<td>Validation Data</td>
<td>170</td>
<td>99</td>
<td>36</td>
<td>30</td>
<td>111</td>
<td>144</td>
</tr>
</tbody>
</table>
Figure 6. The diagram of model validation

Figure 7. The relative error distribution of the validated data

Table 3. The antecedent parameters of the prediction model

<table>
<thead>
<tr>
<th>Parameters Rules</th>
<th>Parameters</th>
<th>(c)</th>
<th>(\sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R^1)</td>
<td>(A_1)</td>
<td>3.824</td>
<td>765.115</td>
</tr>
<tr>
<td></td>
<td>(A_2)</td>
<td>765.115</td>
<td>1.352</td>
</tr>
<tr>
<td>(R^2)</td>
<td>(A_1)</td>
<td>3.788</td>
<td>766.553</td>
</tr>
<tr>
<td></td>
<td>(A_2)</td>
<td>766.553</td>
<td>1.351</td>
</tr>
<tr>
<td>(R^3)</td>
<td>(A_1)</td>
<td>3.784</td>
<td>765.345</td>
</tr>
<tr>
<td></td>
<td>(A_2)</td>
<td>765.345</td>
<td>1.347</td>
</tr>
<tr>
<td>(R^4)</td>
<td>(A_1)</td>
<td>3.802</td>
<td>768.621</td>
</tr>
<tr>
<td></td>
<td>(A_2)</td>
<td>768.621</td>
<td>1.328</td>
</tr>
<tr>
<td>(R^5)</td>
<td>(A_1)</td>
<td>3.800</td>
<td>764.398</td>
</tr>
<tr>
<td></td>
<td>(A_2)</td>
<td>764.398</td>
<td>1.358</td>
</tr>
<tr>
<td>(R^6)</td>
<td>(A_1)</td>
<td>3.974</td>
<td>771.196</td>
</tr>
<tr>
<td></td>
<td>(A_2)</td>
<td>771.196</td>
<td>1.312</td>
</tr>
</tbody>
</table>

Table 4. The consequent parameter of the prediction model

<table>
<thead>
<tr>
<th>Parameters Rules</th>
<th>(p_0)</th>
<th>(p_1)</th>
<th>(p_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R^1)</td>
<td>0.0000069622</td>
<td>0.0000395049</td>
<td>0.0053502312</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.0000069023</td>
<td>0.0000393197</td>
<td>0.0055912311</td>
</tr>
<tr>
<td>(R^3)</td>
<td>0.0000069350</td>
<td>0.0000390175</td>
<td>0.0056326241</td>
</tr>
<tr>
<td>(R^4)</td>
<td>0.0000068916</td>
<td>0.0000388331</td>
<td>0.0053463082</td>
</tr>
<tr>
<td>(R^5)</td>
<td>0.0000069619</td>
<td>0.0000391252</td>
<td>0.0050837386</td>
</tr>
<tr>
<td>(R^6)</td>
<td>0.0000070140</td>
<td>0.0000396185</td>
<td>0.0062976439</td>
</tr>
</tbody>
</table>

According to the calculate result, the model gets a higher precision after we do the trim. So the model will use the result after doing the trim. Then the antecedent parameters and the consequent parameters can be indicated by table 3 and table 4.

Then, this double clustering model can be described as follows:

\[ R^1: \quad \text{if } \Delta t \text{ is } A_1, P_{con} \text{ is } A_2^1 \]

\[ \text{then } COP^1 = 0.0000069622 + 0.0000395049 \Delta t + 0.0053502312 P_{con} \]

\[ (8) \]

\[ R^2: \quad \text{if } \Delta t \text{ is } A_1^1, P_{con} \text{ is } A_2^2 \]

\[ \text{then } COP^2 = 0.0000069023 + 0.0000393197 \Delta t + 0.0055912311 P_{con} \]

\[ (9) \]

\[ R^3: \quad \text{if } \Delta t \text{ is } A_1^2, P_{con} \text{ is } A_2^3 \]

\[ \text{then } COP^3 = 0.0000069350 + 0.0000390175 \Delta t + 0.0056326241 P_{con} \]

\[ (10) \]

\[ R^4: \quad \text{if } \Delta t \text{ is } A_1^3, P_{con} \text{ is } A_2^4 \]

\[ \text{then } COP^4 = 0.0000068916 + 0.0000388331 \Delta t + 0.0053463082 P_{con} \]

\[ (11) \]

\[ R^5: \quad \text{if } \Delta t \text{ is } A_1^4, P_{con} \text{ is } A_2^5 \]

\[ \text{then } COP^5 = 0.0000069619 + 0.0000391252 \Delta t + 0.0050837386 P_{con} \]

\[ (12) \]

\[ R^6: \quad \text{if } \Delta t \text{ is } A_1^5, P_{con} \text{ is } A_2^6 \]

\[ \text{then } COP^6 = 0.0000070140 + 0.0000396185 \Delta t + 0.0062976439 P_{con} \]

\[ (13) \]

Now, self-check should be done for the model by using the identification data, the result is shown in figure 8.
According to the result, the average relative error between model and the sample is 9.98% (maximum relative error is 36.27%, minimum relative error is 0%), and PER is 0.4997. Compare this result with the identification result we got previous, the result is shown in figure 9.

We can find in this figure, that 90% results calculated by model has a good relative error below 20% with the sample data. After using the new data to verify the model, the precision is reduced. But the temperature difference between the average relative error of model identification and model self-check is below 4%, and 75% data has a relative error in 20%, so this model is good enough to use. Next part we will do the feature analysis on the result of model identification and discuss the feasibility of this model.

THE FEASIBILITY OF T-S PREDICTION MODEL

When the T-S Model is applied to the prediction control, first thing to do is to calculate the COP predicted value of operating condition and operating consecutive points, and determine the optimal variation trend by making a contrast among these points. Then determine the control strategy of the optimal variation trend. So from the perspective of the application, the prediction precision of operating consecutive points is much more important than the absolute precision of model.

At first, make an analysis on the variation of relative error varies with the input variable. From the result of the analysis, we can see that the identification data are equally distributed when the relative error is below 20%, that means the model inputs have less impact on the relative error. When the relative error is exceed 20%, the data mainly focus on the range of temperature difference around 2~3 ℃ and condensing pressure around 690~770kPa, it says the model is not good in this data range.

For further analysis on the prediction precision, next we will dwell on the different relative error sections of identification data and validation data. Figure 10–14 respectively are temperature difference vs pressure chart with the relative error range among below 5%, 5%~10%, 10%~15%, 15%~20% and exceed 20%. In the chart, square point means the value is positive and round point means the value is negative.

Now, select the operating condition’s reference point for each relative error range and analyze the data in which the condensing pressure and temperature difference respectively varies ±25kpa and ±0.5℃ from the base point. If the difference of COP in actual has the same sign to the calculated COP, then the trend prediction is right. The trend prediction result can be seen in table 5. From the statistical result, we can find that the prediction precision of identification data for each relative error range is exceed 80%, except the error range in 5%~10%. For the validation data, the prediction precision for relative error range respectively in 5%~10%, 15%~20% and exceed 20% is below 80%, it may be caused by the shortage of the identification data. But whatever it is for identification data or for validation data, the prediction precision for all of the relative error ranges can exceed 50%, that means the model can predict the COP trend for most operating conditions.

Take an analysis on the application of model, for both identification data and validation data, 75% of the prediction results have a good relative error within 20%. When the model calculation relative error exceeds 20%, the T-S model can perfect predict the COP trend for 68% of the operating conditions. So for both the model precision and the trend prediction precision, the T-S fuzzy model is feasible for optimal control of chiller plants.

![Figure 10. Temperature difference vs condensing pressure with the relative error below 5% (the left is identification data, the right is validation data)](image1)

![Figure 11. Temperature difference vs condensing pressure with the relative error in 5%~10% (the left is identification data, the right is validation data)](image2)

![Figure 12. Temperature difference vs condensing pressure with the relative error in 10%~15% (the left is identification data, the right is validation data)](image3)

![Figure 13. Temperature difference vs condensing pressure with the relative error in 15%~20% (the left is identification data, the right is validation data)](image4)

![Figure 14. Temperature difference vs condensing pressure with the relative error exceed 20% (the left is identification data, the right is validation data)](image5)
### Table 5. The statistical table of prediction results on COP trend

<table>
<thead>
<tr>
<th>Error range</th>
<th>Items</th>
<th>Base point</th>
<th>Identification data</th>
<th>Validation data</th>
<th>Prediction precision ratio</th>
<th>Data size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Identification data</td>
<td>Validation data</td>
<td>Identification data</td>
<td>Validation data</td>
</tr>
<tr>
<td>&lt;5%</td>
<td></td>
<td></td>
<td>340</td>
<td>94</td>
<td>(85.39%)</td>
<td>(87.74%)</td>
</tr>
<tr>
<td>5%~10%</td>
<td></td>
<td></td>
<td>164</td>
<td>62</td>
<td>(68.49%)</td>
<td>(68.54%)</td>
</tr>
<tr>
<td>10%~15%</td>
<td></td>
<td></td>
<td>232</td>
<td>90</td>
<td>(84.00%)</td>
<td>(84.76%)</td>
</tr>
<tr>
<td>15%~20%</td>
<td></td>
<td></td>
<td>154</td>
<td>101</td>
<td>(92.17%)</td>
<td>(66.67%)</td>
</tr>
<tr>
<td>&gt;20%</td>
<td></td>
<td></td>
<td>97</td>
<td>93</td>
<td>(80.67%)</td>
<td>(68.15%)</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

For the optimal control of the chiller plants, this paper introduces the double clustering T-S fuzzy model which can predict the system COP perfectly. The conclusion includes:

- The double clustering T-S model is a good prediction model with obvious advantages, which can use the operating data we’ve known to predict the energy consume.
- After the first clustering, the diversity among clusters are getting obvious, that means the condensing pressure has much more impact on COP.
- From the prediction results we can find that more than 75% of the results get the relative error below 20%, the prediction precision is pretty good.
- Select the operating condition’s reference point for each relative error range, and analyze the data in which the condensing pressure and temperature difference vary in a certain range. We can find in the result that the model can perfect predict the trend for more than 66% of neighboring operating conditions.

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**REFERENCES**